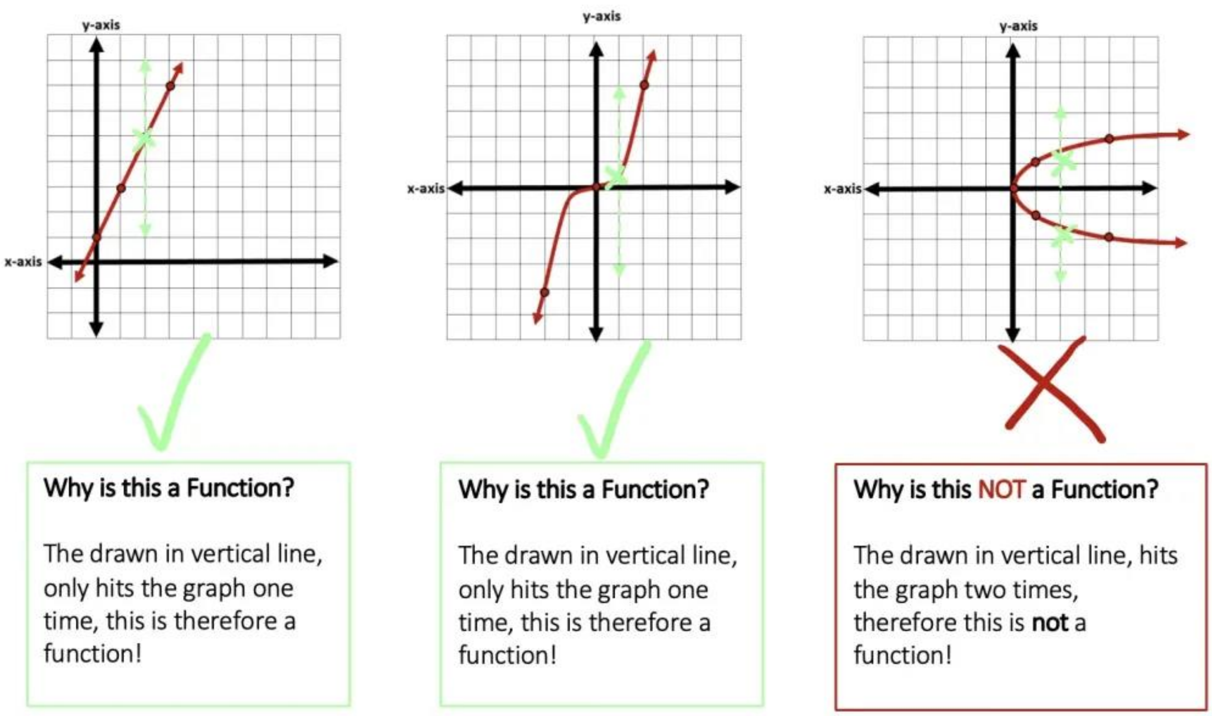
Functions

Functions:

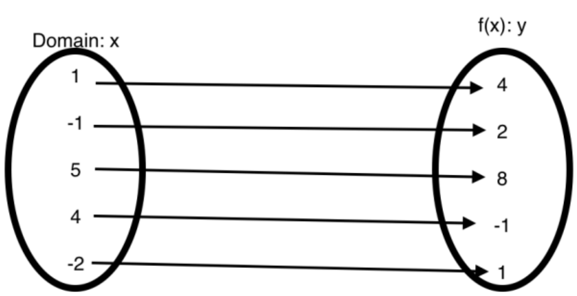
In mathematics, a function is a relation between a set of inputs (called the domain) and a set of possible outputs (called the codomain), where each input is related to exactly one output. In other words, a function assigns each input exactly one output.

How to test if it is a function or not:

Vertical line test:



Domain and range test:



Since there is only one output (y value) for each input (x value), it is a function. It won't be a function when there is more than one output for any input.

Input and output equations:

In a function equation, the input is the value which is added to the operation and the resultant value is the output of the function. The value which goes into the function will produce a value which is the output of the function. Usually the input looks like this: f(5), in this case the input value to the variable in the equation of the function was 5. Here are some example equations:

$$f(x) = 5x + 2$$

$$f(3) = 5(3) + 2 = 17 \rightarrow f(3) = 17$$

$$f(5) = 5(5) + 2 = 27 \rightarrow f(5) = 27$$

$$f(y) = y^2 - 2$$

$$f(4) = (4)^2 - 2 = 14 \rightarrow f(4) = 14$$

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 2 = -1.75 \rightarrow f\left(\frac{1}{2}\right) = -1.75$$

Composite functions:

Composite functions operate with two different functions which are related to each other. Composite functions basically mean the input to a function is the output of another function. For example, gf(x) -> this means that the input which will go into the equation of g(x) will be the output of f(x). Here are some examples:

$$g(x) = 5x + 2, f(x) = 2x - 2$$

$$gf(x) = 5(2x - 2) + 2 = 10x - 10 + 2 = 10x - 8$$

$$gf(8) = 10(8) - 8 = 72 \rightarrow gf(8) = 72$$

or

$$gf(8) \rightarrow 2(8) - 2 = 14 \rightarrow 5(14) + 2 = 72 \rightarrow gf(8) = 72$$

$$g(x) = 4x + 2, f(x) = x^2 + 32$$

$$gg(x) = 4(4x + 2) + 2 = 16x + 8 + 2 = 16x + 10 \rightarrow gg(x) = 16x + 10$$

$$fg(x) = (4x + 2)^2 + 32 = 16x^2 + 16x + 4 + 32 = 16x^2 + 16x + 36$$

$$fg(3) = 16(3)^2 + 16(3) + 36 = 228 \rightarrow fg(3) = 228$$

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$ff(x) = \left(x^2 + 32\right)^2 + 32 = x^4 + 64x^2 + 1024 + 32 = x^4 + 64x^2 +$ [redacted]

Inverse functions:

Inverse functions are when you take the opposite function of the expression in the function. It is denoted as f to the power –1 of x. You can find the inverse of the function by replacing all the x variables in the function with y and the y variable with x. Then you will solve for y to make the inverse function. Here are some examples:

$$f(x) = x^2 + 5, g(x) = \frac{(2x + 4)}{4}$$

$$g^{-1}(x) \rightarrow x = \frac{(2y + 4)}{4} \rightarrow \frac{(4x - 4)}{2} = y \rightarrow g^{-1}(x) = \frac{(4x - 4)}{2}$$

$$f^{-1}(x) \rightarrow x = y^2 + 5 \rightarrow \sqrt{x} - 5 = y \rightarrow f^{-1}(x) = \sqrt{x} - 5$$

$$s(c) = 24c - 2$$

$$s^{-1}(c) \rightarrow c = 24s - 2 \rightarrow \frac{(c + 2)}{24} = s \rightarrow s^{-1}(c) = \frac{(c + 2)}{24}$$

Complex function equations:

$$f(x) = x^2 + 4x - 2$$

$$f(5x) = (5x)^2 + 5(5x) - 2 = 25x^2 + 25x - 2$$

$$g(x) = x^2 - 2$$

$f(x) = x^2 - 2x$

$f(3x) - f(x-1) = 4$

$f(3x) = (3x)^2 - 2(3x) = 9x^2 - 6x$

$f(x-1) = (x-1)^2 - 2(x-1) = x^2 - 4x + 3$

$9x^2 - 6x - \left(x^2 - 4x + 3\right) = 4$

$x = 1.07, \, or \; = \; -0.82$

$f(x) = \frac{\left(x^2 - 2\right)}{4}$

$f^{-1}(4) = ?$

$f^{-1}(x) \to x = \frac{\left(y^2 - 2\right)}{4} \to 4x = y^2 - 2 \to 4x + 2 = y^2$

$y = \sqrt{4x+2} \to f^{-1}(x) = \sqrt{4x+2}$

$f^{-1}(4) = \sqrt{4(4)+2} = \pm 4.24$

$f(x) = x + 3, \, g(x) = x^2 + 25$

$solve \; gf(x) = 0$

$(x+3)^2 - 25 = x^2 + 6x + 9 - 25 = x^2 + 6x - 16$

$x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$

$x = -8, \, or \; x = 2$

$$f(x) = x^3 - 5x, g(x) = 3x + 5$$

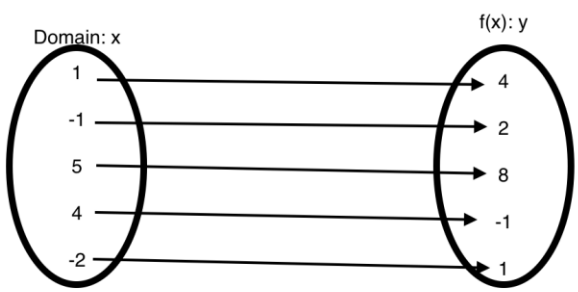
$$gf(x) = 3(x^3 - 5x) + 5 = 3x^3 - 15x + 5$$

$$gf(x) = (3x + 5)^3 - 5x = 27x^3 - 135x^2 + 225x - 125 - 5x$$

$$gf(x) = 27x^3 - 135x^2 + 220x - 125$$

Domain and Range:

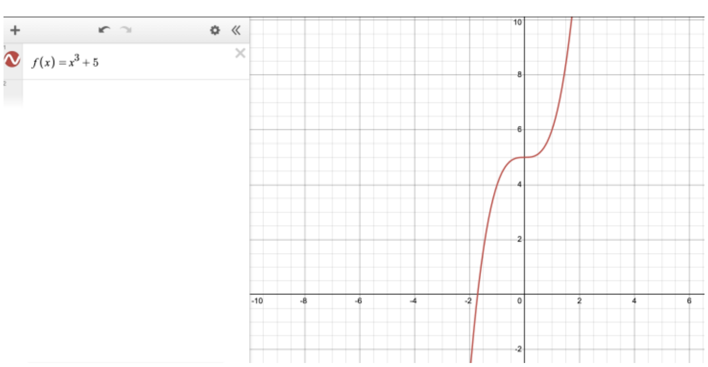
Domains are all the possible values of x that can be inputted into the function. On the other hand, range are all the y-values which are the outputs from the function. These two can be written in terms of inequalities to explain the domain and range of a function.

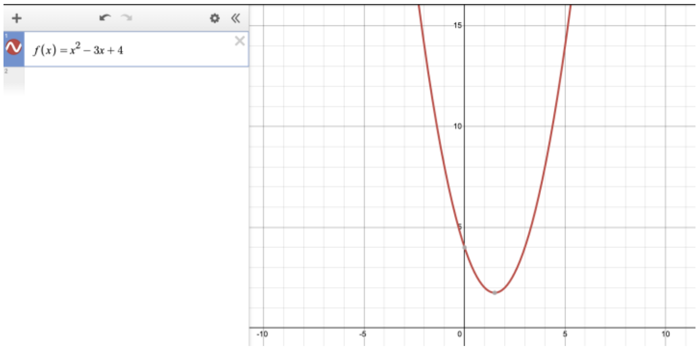


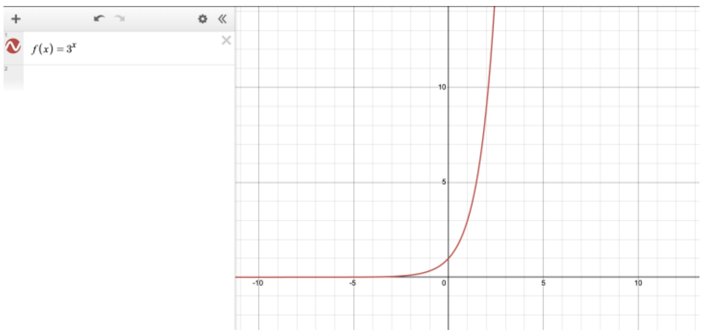
The domain in this example would be the values for x which are applicable to the function (on the left). The range of this function would be all the output values for y from the function (the ones on the right).

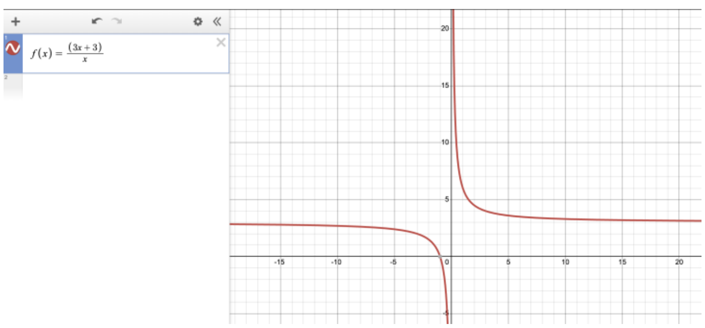
Function graphs:

Graphing functions are like graphing any expression. Here are some examples:





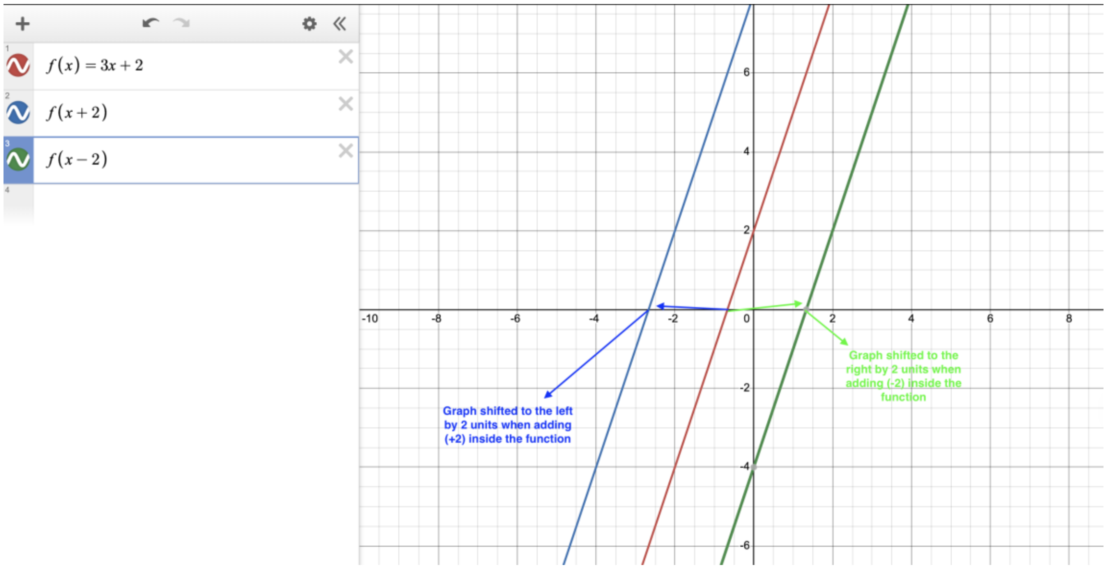




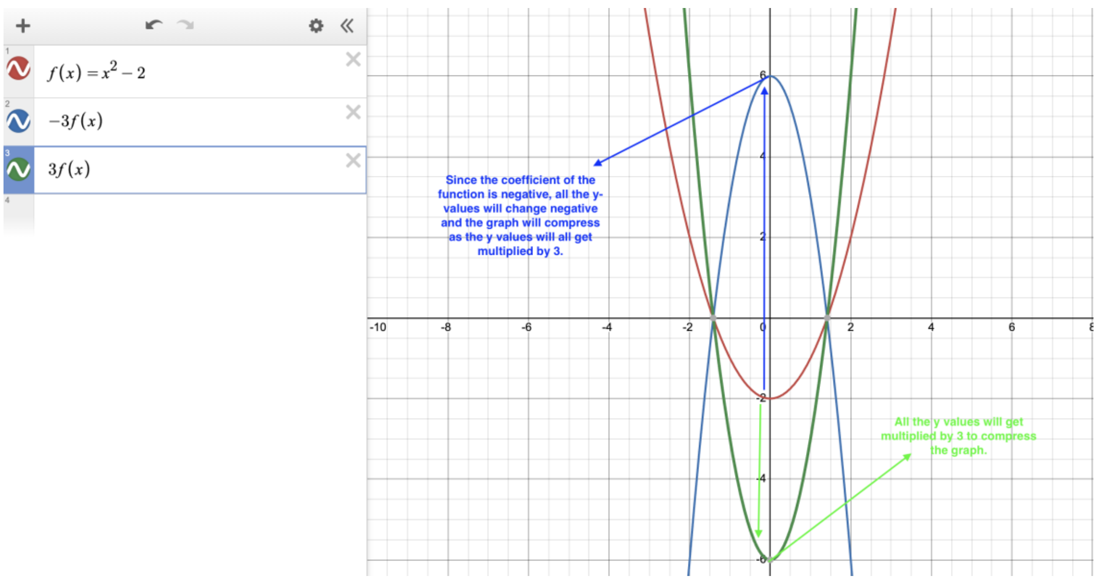
Graph transformations:

There are many properties that can change or affect the graph of a function. These properties will be explained in each image.

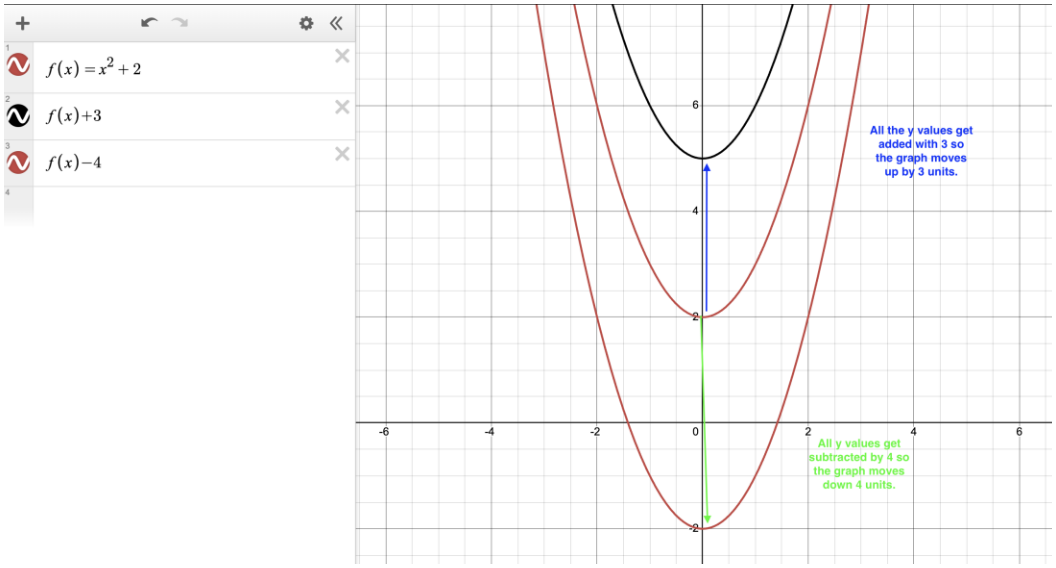
1.) Units inside the function



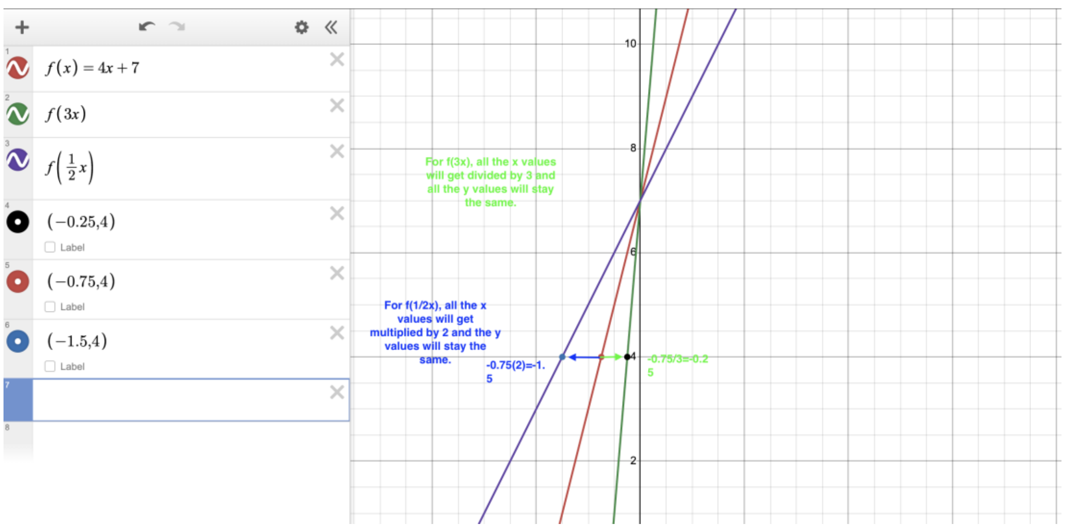
2.) Units outside the function



3.) units on the right of the function

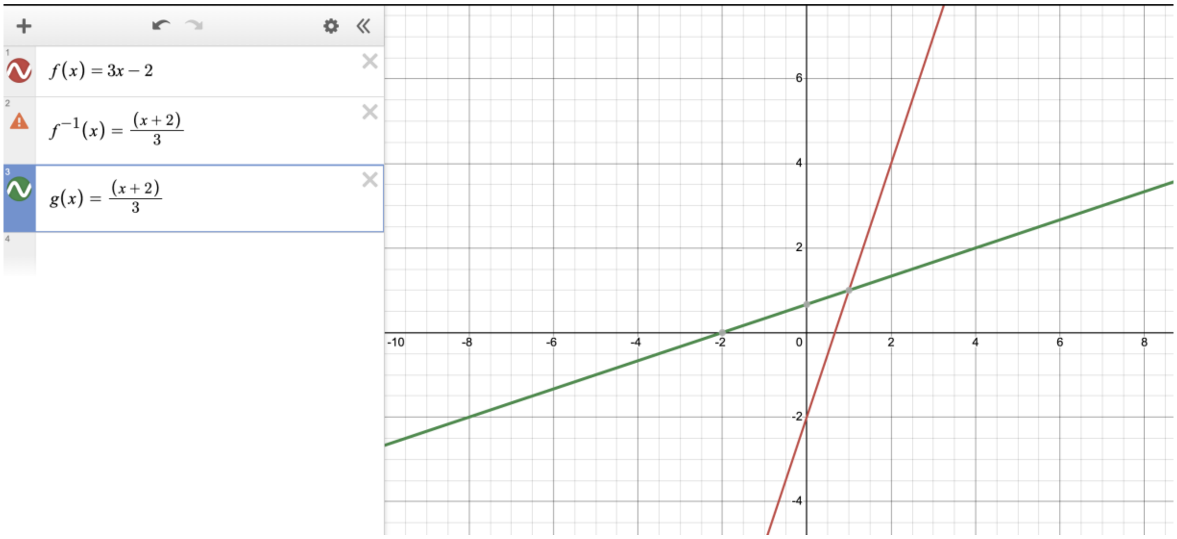


4.) Multiplication in the function



Inverse graphs:

Inverse graphs are when you find the inverse function and then plot the graph of that expression.



$f(x) = 3x - 2$
$f^{-1}(x) = \frac{(x+2)}{3}$
$g(x) = \frac{(x+2)}{3}$
-10
-8
-6
-4
-2
0
2
4
6
8
6
4
2
-2
-4